Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Multiple Terms and Conjugates

Sometimes it is necessary to multiply or divide radical expressions with more than one *term*. To multiply radicals with multiple terms by a single term, we use the old reliable **distributive property**. See how the *distributive property* works for these examples.

Example 1

$$6(\sqrt{5} + \sqrt{3}) =$$
$$6\sqrt{5} + 6\sqrt{3}$$

Example 2

$$
\begin{aligned}
&\sqrt{3}(2\sqrt{5} - 4\sqrt{3}) = \\
&\quad 2\sqrt{15} - 4\sqrt{9} = \\
&\quad 2\sqrt{15} - 4 \bullet 3 = \\
&\quad 2\sqrt{15} - 12
\end{aligned}
$$

Example 3

$$
\begin{aligned}
&6\sqrt{3}(2\sqrt{2} + 5\sqrt{6}) = \\
&12\sqrt{6} + 30\sqrt{18} = \\
&12\sqrt{6} + 30\sqrt{9}\sqrt{2} = \\
&12\sqrt{6} + 30 \bullet 3\sqrt{2} = \\
&12\sqrt{6} + 90\sqrt{2}
\end{aligned}
$$

The FOIL Method

Another reliable method we can use when multiplying two radical expressions with multiple terms is the **FOIL method:** multiplying the **f**irst, **o**utside, **i**nside, and **l**ast terms. We use that same process in problems like these.

Example 1

O F I L

$$(\sqrt{6} - 5)(\sqrt{3} + 4) =$$

F O I L

$$\sqrt{6} \bullet \sqrt{3} + \sqrt{6} \bullet 4 - 5 \bullet \sqrt{3} - 5 \bullet 4 =$$

← Multiply the **first** terms, the **outside** terms, the **inside** terms, and then the **last** terms.

$$\sqrt{18} + 4\sqrt{6} - 5\sqrt{3} - 20 =$$

← Carefully write out the **products**.

$$3\sqrt{2} + 4\sqrt{6} - 5\sqrt{3} - 20$$

← Simplify each term and combine like terms (if needed).

Example 2

$$(\sqrt{3} + \sqrt{2})(\sqrt{7} - \sqrt{11}) =$$

$$\sqrt{3}\sqrt{7} - \sqrt{3}\sqrt{11} + \sqrt{2}\sqrt{7} - \sqrt{2}\sqrt{11} =$$

$$\sqrt{21} - \sqrt{33} + \sqrt{14} - \sqrt{22}$$

← Notice that no term has a perfect square as a factor. Therefore, there is no further simplifying to be done.

Time to try the following practice.

Two-Term Radical Expressions

At the beginning of this unit, we learned that there are two rules we must remember when simplifying a radical expression. Rule one requires that we never leave a perfect square factor under a radical sign. Rule two insists that we never leave a radical in the denominator. With that in mind, let's see what to do with two-term radical expressions.

In a problem like $\frac{2 + \sqrt{7}}{5 - \sqrt{6}}$, we see that we must rationalize the denominator (reformat it without using a square root). At first glance, it may seem to you that multiplying that denominator by itself makes the square roots disappear. But when we try that, we realize that new square roots appear as a result of the FOILing.

$$(5 - \sqrt{6})(5 - \sqrt{6}) =$$

$$25 - 5\sqrt{6} - 5\sqrt{6} + \sqrt{6}\sqrt{6} =$$

$$25 - 10\sqrt{6} + 6$$

So there must be a better way to rationalize this denominator. Try multiplying ($5 - \sqrt{6}$) by its **conjugates** ($5 + \sqrt{6}$). These numbers are *conjugates* because they match, except for the signs between the terms. Notice that one has a "+" and the other has a "–".

$$(5 - \sqrt{6})(5 + \sqrt{6}) =$$

$$25 + 5\sqrt{6} - 5\sqrt{6} - \sqrt{6}\sqrt{6} =$$

$$25 - \sqrt{36} =$$

$$25 - 6 =$$

$$19$$

Remember, we only need to rationalize the denominator. It is acceptable to leave simplified square roots in the numerator. Now, let's take a look at the entire problem.

$$\frac{2+\sqrt{7}}{5-\sqrt{6}} \bullet \frac{5+\sqrt{6}}{5+\sqrt{6}} =$$ ⟵ reformat the fraction by multiplying it by 1

$$\frac{5+\sqrt{6}}{5+\sqrt{6}} = 1$$

$$\frac{(2)(5)+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{(5)(5)+5\sqrt{6}-5\sqrt{6}-\sqrt{6}\sqrt{6}} =$$ ⟵ FOIL the numerator and denominator

$$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{25-\sqrt{36}} =$$ ⟵ simplify

$$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{25-6} =$$ ⟵ simplify again

$$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{19}$$ ⟵ and again, if necessary

Follow along with this one!

$$\frac{3+\sqrt{2}}{4+\sqrt{8}} \bullet \frac{4-\sqrt{8}}{4-\sqrt{8}} =$$ ⟵ reformat the fraction by multiplying it by 1 $\frac{4-\sqrt{8}}{4-\sqrt{8}} = 1$

$$\frac{(3)(4)-3\sqrt{8}+4\sqrt{2}-\sqrt{16}}{(4)(4)-4\sqrt{8}+4\sqrt{8}-\sqrt{8}\sqrt{8}} =$$ ⟵ FOIL the numerator and denominator

$$\frac{12-3\sqrt{4}\sqrt{2}+4\sqrt{2}-\sqrt{16}}{16-\sqrt{64}} =$$ ⟵ simplify

$$\frac{12-3 \bullet 2\sqrt{2}+4\sqrt{2}-4}{16-8} =$$ ⟵ simplify again

$$\frac{12-6\sqrt{2}+4\sqrt{2}-4}{8} =$$ ⟵ and again

$$\frac{8-2\sqrt{2}}{8} = \frac{2(4-\sqrt{2})}{8} =$$ ⟵ and again

$$\frac{4-\sqrt{2}}{4}$$ ⟵ and again, if necessary

With more practice, you will be able to mentally combine some of those simplifying steps and finish sooner.

So let's practice on the following page.